Dark energy on cosmological and astrophysical scales: predictions and perspective of testing

M. Tsizh, S. Apunevych, Yu. Kulinich, B. Novosyadlyj
Ivan Franko National University of Lviv, Ukraine

Ukrainian-Italian workshop
Kharkiv, September, 2018

Definition of dark energy

The physical essence which is causing the accelerated expansion of the Universe which is described in the framework of the general relativity (GR):

$$g(r) = -\frac{4\pi}{3}G(\rho + 3p/c^2)r > 0,$$

$$\rho + 3p/c^2 = \rho_m + 3p_m/c^2 + \rho_X + 3p_X/c^2 < 0,$$

$$p_X < -\frac{1}{3}c^2(\rho_m + \rho_X) - p_m$$

Component X have been called the dark energy (Huterer D. & Turner M. 1998).

Source of gravitational field: $c^2\rho + 3p = c^2\rho(1 + 3w)$

Inertial mass: $c^2\rho + p = c^2\rho(1 + w)$

Observational evidence for existence of dark energy

- apparent magnitude - redshift for SNe Ia,
- apparent magnitude - redshift for GRBs,
- acoustic peaks in the angular power spectrum of the CMB,
- baryon acoustic oscillations in the spatial distribution of galaxies,
- angular size - redshift for X-ray galaxy clusters,
- formation of the large scale structure of the Universe,
- cross-correlation of ISW effect for CMB and the spatial distribution of galaxies,
- weak gravitational lensing of CMB,
- age of oldest stars in the Galaxy.

Key experiments of 1998-2018 years

1998 – HzSNST (SN Ia, *Riess et al.*)

1998 – SNCP (SN Ia, *Perlmutter et al.*)

1999 – Toco (CMB, *Miller et al.*)

2000 – Boomerang (CMB, *Bernardis etal*)

2000 – MAXIMA (CMB, *Hanany et al.*)

2001 – DASI (CMB, *Halverson et al.*)

2003 – WMAP-1 (CMB, *Spergel et al.*)

2005 – BAO (SDSS, *Eisenstein et al.*)

2006 – WMAP-3 (CMB, *Spergel et al.*)

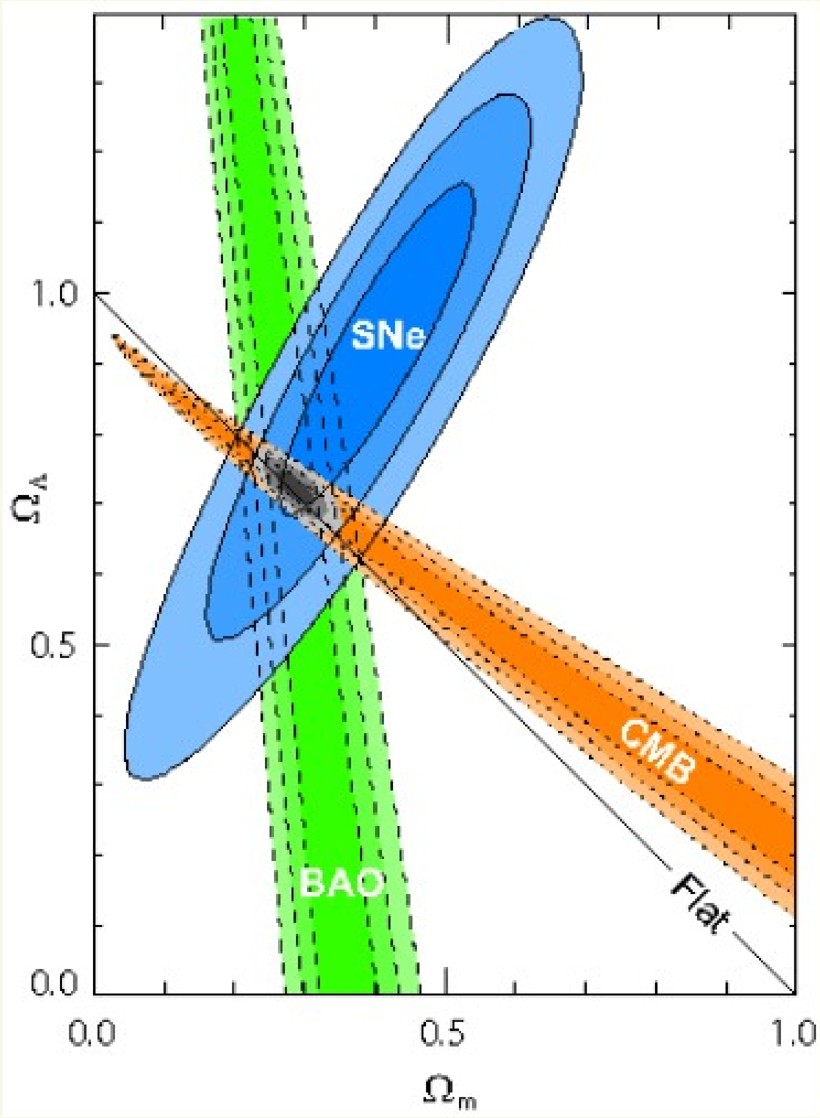
2008 – WMAP-5 (CMB, *Komatsu et al.*)

2011 – WMAP-7 (CMB, *Komatsu et al.*)

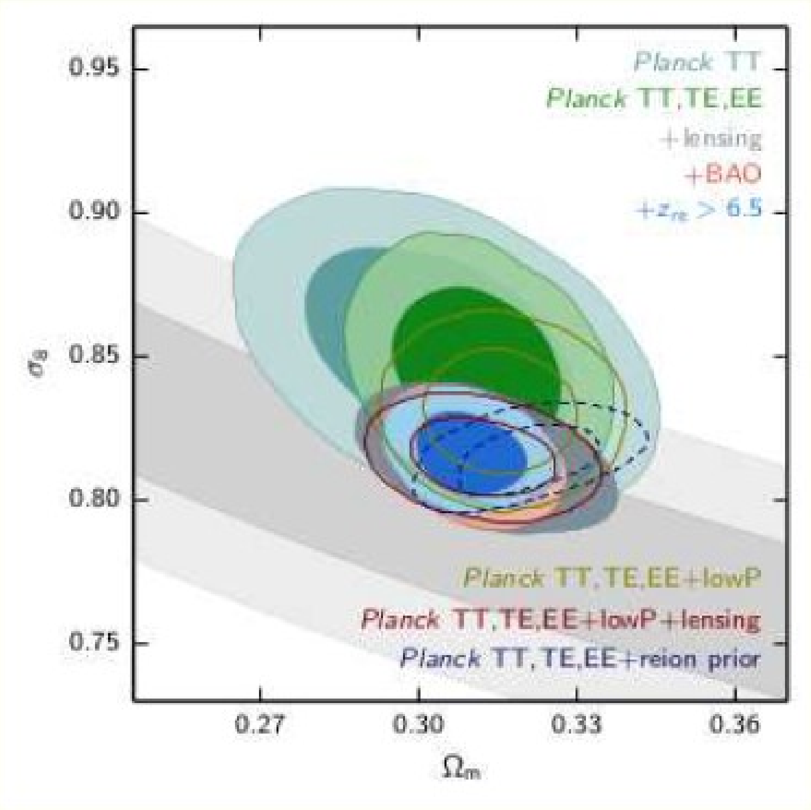
2011 – ACT (CMB, *Dunkley et al.*)

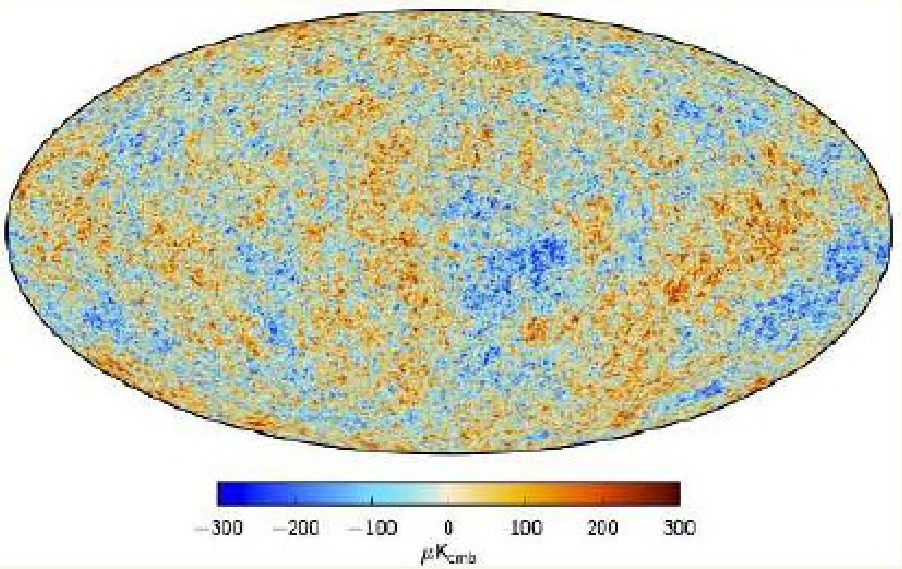
2013 – WMAP (CMB, *Bennett et al.*)

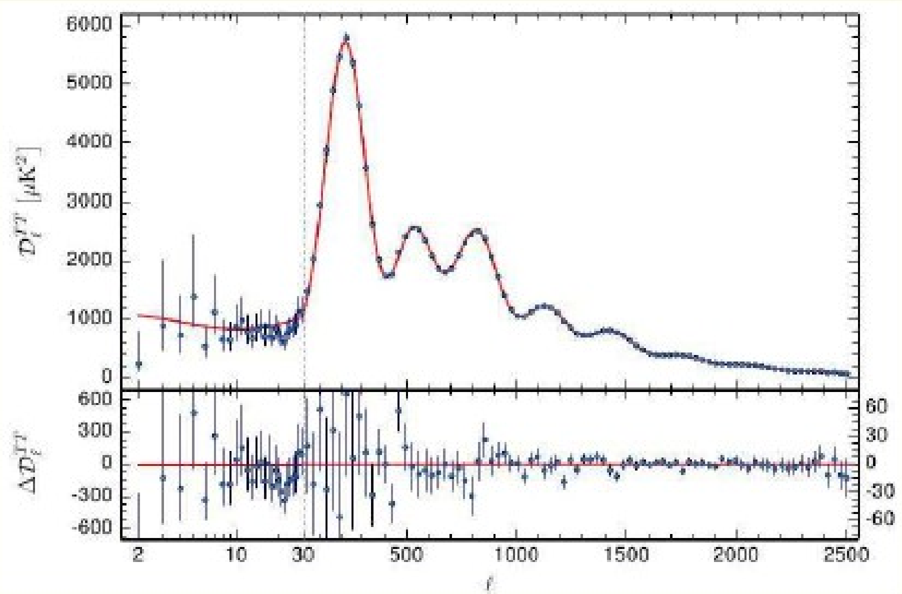
2018 – Planck (CMB, *Planck Collaboration*)



Temperature fluctuations of CMB







The density parameter of dark matter determined from the Plank2015 data is as follows ($H_0 = 68.7$ km/s/Mpc):

$$\Omega_{dm} = 0.251 \pm 0.004, \quad \Omega_b = 0.0484 \pm 0.001$$

Planck Collaboration, Planck 2015 results.I., A&A 594, A1 (2016)

Cosmological parameters from Planck 2018 (18.07.2018)

Parameters	Planck alone	Planck +BAO
$\Omega_b h^2$	0.02237± 0.00015	0.02242± 0.00014
$\Omega_{dm} h^2$	0.1200± 0.0012	0.11933± 0.00091
$100\theta_{MC}$	1.04092± 0.00031	1.04101± 0.00029
τ	0.0544± 0.0073	0.0561± 0.0071
$\ln(10^{10} A_s)$	3.044± 0.014	3.047± 0.014
n_s	0.9649± 0.0042	0.9665± 0.0038
H_0	67.36± 0.54	67.66± 0.42
Ω_Λ	0.6847± 0.0073	0.6889± 0.0056
Ω_m	0.3153± 0.0073	0.3111± 0.0056
σ_8	0.8111± 0.0060	0.8102± 0.0060
z_{re}	7.67±0.73	7.82±0.71
Age[Gyr]	13.797± 0.023	13.787± 0.020

Planck Collaboration, Planck 2018 results.I., arXiv:1807.06205 (2018)

Candidates for dark energy

- cosmological constant Λ,
- scalar field (quintessence, phantom, quintom, K-essence, tachyon field, Chaplygin gas, barotropic fluid ...) which almost homogeneously fills the Universe,
- more general theory than GR or another gravitation theory (Brans-Dicke theory, f(R)-gravity, dilaton gravity, MOND...) .

Scalar field $\Longleftrightarrow$ dynamical fluid

$$\mathcal{L}(X, U(\phi)), \qquad X \equiv \frac{1}{2}\phi_{;i}\phi^{;i}$$

$$T_{ij} = \mathcal{L}_{,X}\phi_{,i}\phi_{,j} - g_{ij}\mathcal{L}$$

$$T_{ij} = (\rho_{de} + p_{de})u_i u_j - p_{de} g_{ij}$$

$$p_{de} = \mathcal{L} \qquad\qquad \rho_{de} = 2X\mathcal{L}_{,X} - \mathcal{L}$$

$$w_{de} \equiv \frac{p_{de}}{\rho_{de}} = \frac{\mathcal{L}}{2X\mathcal{L}_{,X} - \mathcal{L}} < -\frac{1}{3} \qquad c_s^2 \equiv \frac{\delta p_{de}}{\delta \rho_{de}} = \frac{\mathcal{L}_{,X}}{\mathcal{L}_{,X} + 2X\mathcal{L}_{,XX}} \geq 0$$

$$\Omega_{de} \equiv \frac{\rho_{de}}{\rho_{cr}} = \frac{8\pi G}{3H_0^2}\rho_{de} \qquad c_a^2 \equiv \frac{\dot{p}_{de}}{\dot{\rho}_{de}}$$

Dark energy and expansion of the Universe

$$\text{Einstein equations :} \quad R_{ij} - \frac{1}{2} g_{ij} R = \kappa \left(T_{ij}^{(r)} + T_{ij}^{(m)} + T_{ij}^{(de)} \right)$$

$$\text{Friedmann metric :} \quad ds^2 = g_{ij} dx^i dx^j = c^2 dt^2 - a^2(t) \delta_{\alpha\beta} dx^\alpha dx^\beta$$

$$\text{EoS equation :} \quad p = w c^2 \rho \quad (w_r = \frac{1}{3}, \quad w_m = 0, \quad w_{de} < -\frac{1}{3})$$

$$T_{i;k}^{k\,(r)} = 0 \quad \rightarrow \quad \dot{\rho}_r = -4 \frac{\dot{a}}{a} \rho_r \quad \rightarrow \quad \rho_r(t) = \rho_r^0 a^{-4}$$

$$T_{i;k}^{k\,(m)} = 0 \quad \rightarrow \quad \dot{\rho}_m = -3 \frac{\dot{a}}{a} \rho_m \quad \rightarrow \quad \rho_m(t) = \rho_m^0 a^{-3}$$

$$T_{i;k}^{k\,(de)} = 0 \quad \rightarrow \quad \dot{\rho}_{de} = -3(1+w_{de}) \frac{\dot{a}}{a} \rho_{de} \quad \rightarrow \quad \rho_{de}(t) = \rho_{de}^0 a^{-3(1+\tilde{w}_{de})},$$

where

$$\tilde{w}_{de}(a) = \frac{1}{\ln(a)} \int_1^a w_{de}(a') d\ln a' \quad \text{and} \quad \tilde{w}_{de} = w_{de} \quad \text{for} \quad w_{de} = const$$

Specifying of scalar field models of dark energy

$$\frac{\dot{p}_{de}}{\dot{\rho}_{de}} = c_a^2 = const \quad \rightarrow \quad p_{de} = c_a^2 \rho_{de} + C$$

[Babichev, Dokuchaev & Eroshenko (2005)]

$$\frac{dw_{de}}{da} = 3a^{-1}(1 + w_{de})(w_{de} - c_a^2)$$

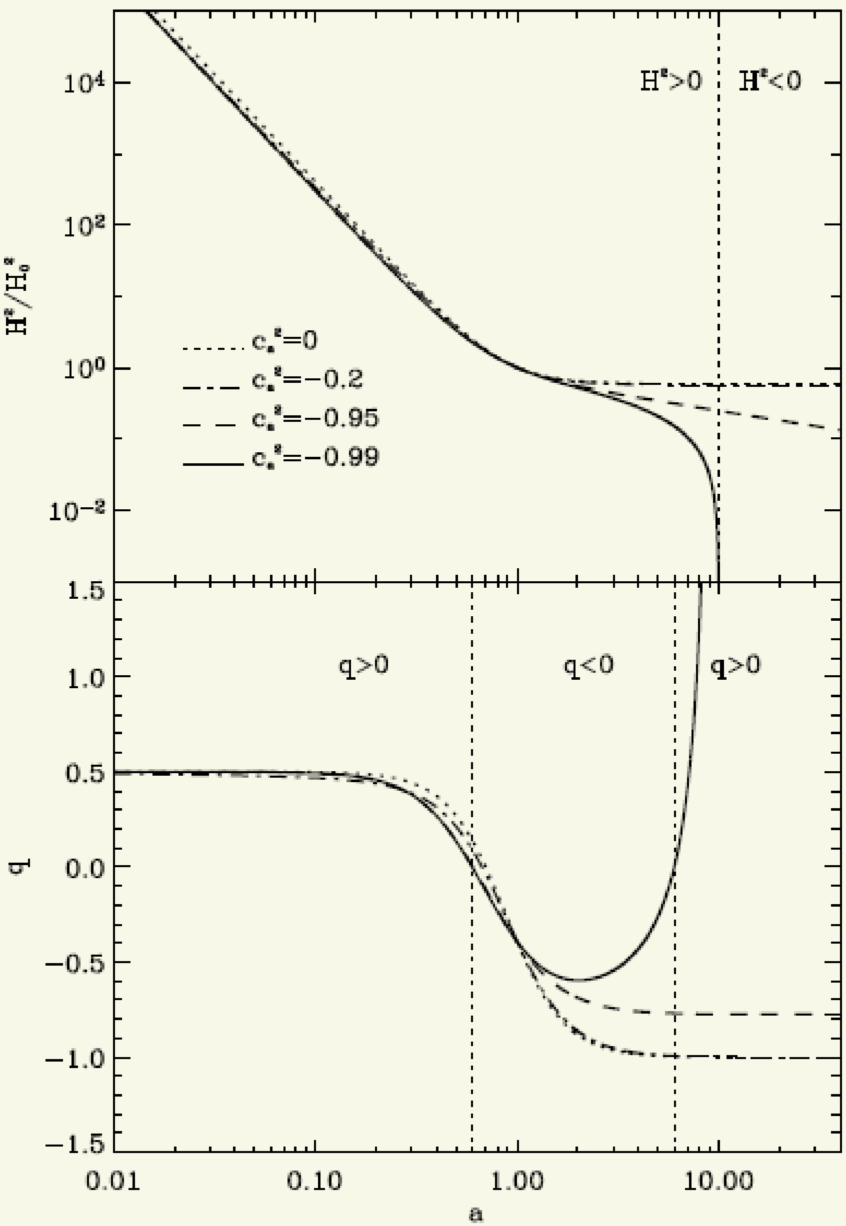
$$w_{de}(a) = \frac{(1 + c_a^2)(1 + w_0)}{1 + w_0 - (w_0 - c_a^2)a^{3(1+c_a^2)}} - 1$$

$$\rho_{de}(a) = \rho_{de}^{(0)} \frac{(1 + w_0)a^{-3(1+c_a^2)} + c_a^2 - w_0}{1 + c_a^2}$$

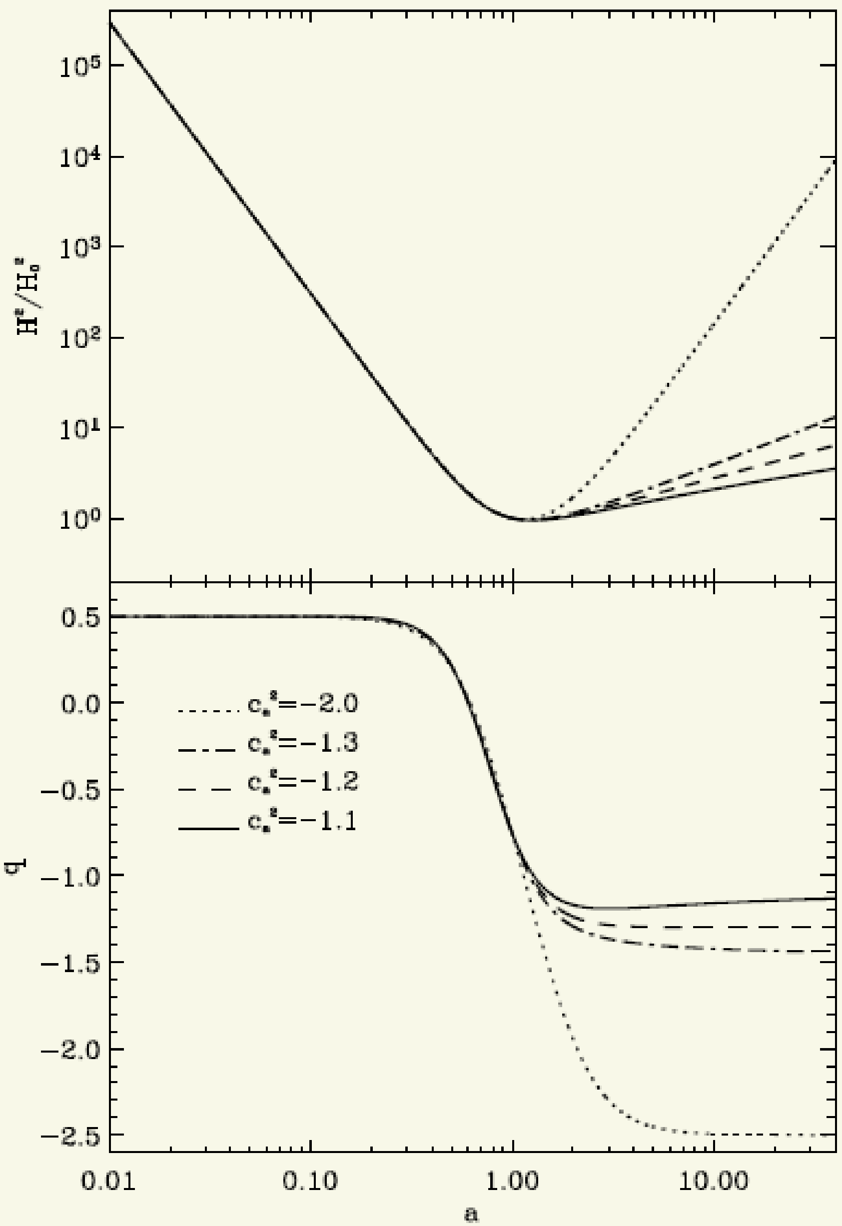
$$f(a) = \frac{(1 + w_0)a^{-3(1+c_a^2)} + c_a^2 - w_0}{1 + c_a^2}$$

Dynamics of expansion of the Universe with scalar field dark energy

Quintessence

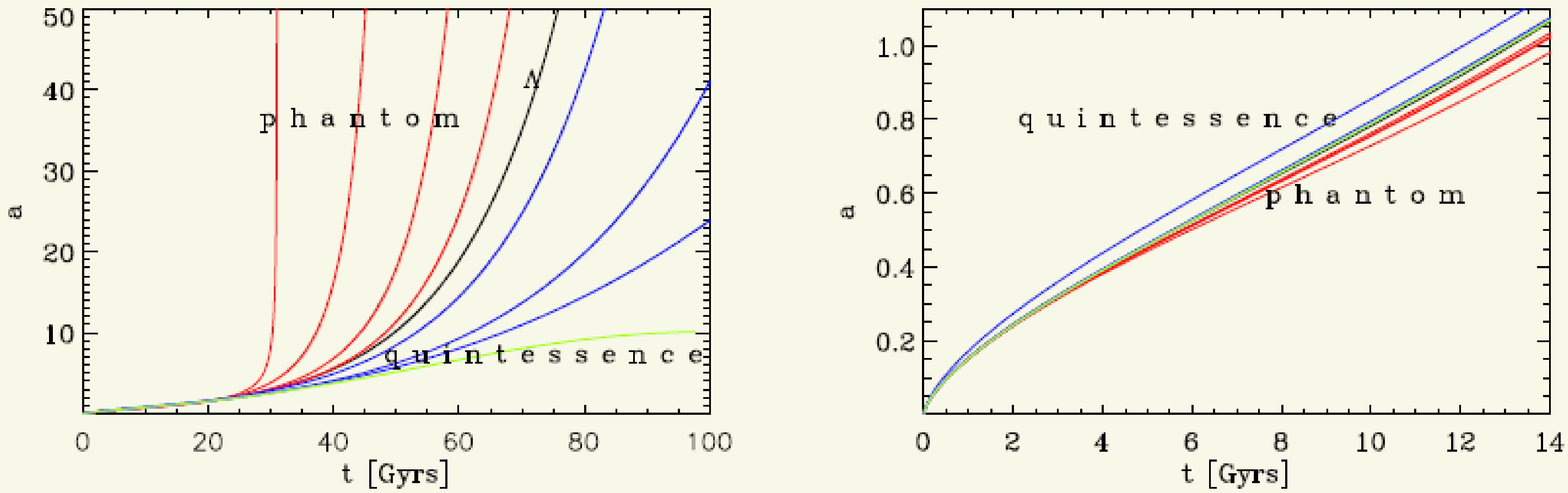


Phantom



Future of the Universe depends on the nature of dark energy

$$t(a) = \int_0^a \frac{da'}{a' H(a')} \quad \rightarrow \quad a(t)$$



Big Rip singularity: $t_{BR} - t_0 \approx \frac{2}{3}\frac{1}{H_0}\frac{1}{|1+c_a^2|}\sqrt{\frac{1+c_a^2}{(1+w_0)\Omega_{de}}}$

Perturbations:

$$\rho = \overline{\rho}(1+\delta), \quad p = \overline{p}(1+\pi), \quad u^i = \overline{u}^i + \delta u^i,$$
$$T_{ij} = \overline{T}_{ij} + \delta T_{ij}$$

$$ds^2 = c^2 dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j \quad (h \equiv h^i_i \ll 1),$$
$$R_{ij} = \overline{R}_{ij} + \delta R_{ij}, \quad R = \overline{R} + \delta R$$

Equations for Fourier modes of perturbations in the synchronous gauge comoving to matter component ($V_m = 0$):

$$\dot{\delta}_{de} + 3(c_s^2 - w_{de})aH\delta_{de} + (1+w_{de})\frac{\dot{h}}{2} + (1+w_{de})\left[k + 9a^2H^2\frac{c_s^2 - c_a^2}{k}\right]V_{de} = 0,$$

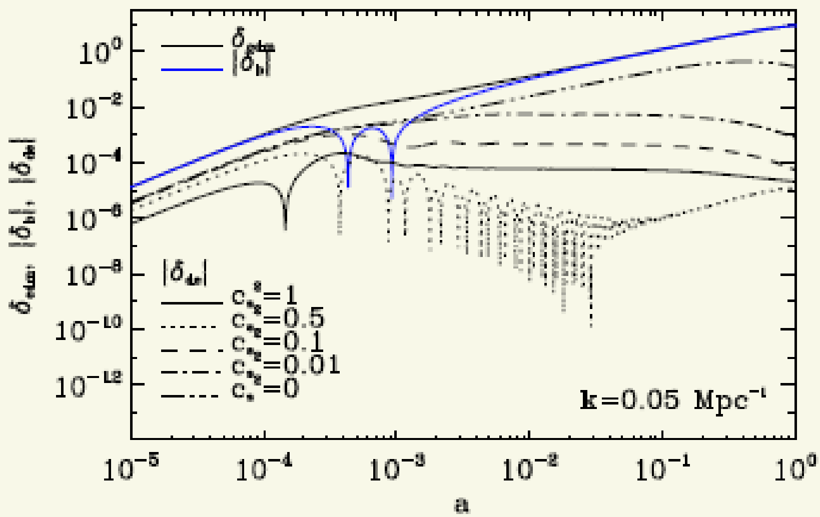
$$\dot{V}_{de} + aH(1-3c_s^2)V_{de} - \frac{c_s^2 k}{1+w_{de}}\delta_{de} = 0,$$

$$\dot{\delta}_m = -\frac{1}{2}\dot{h},$$

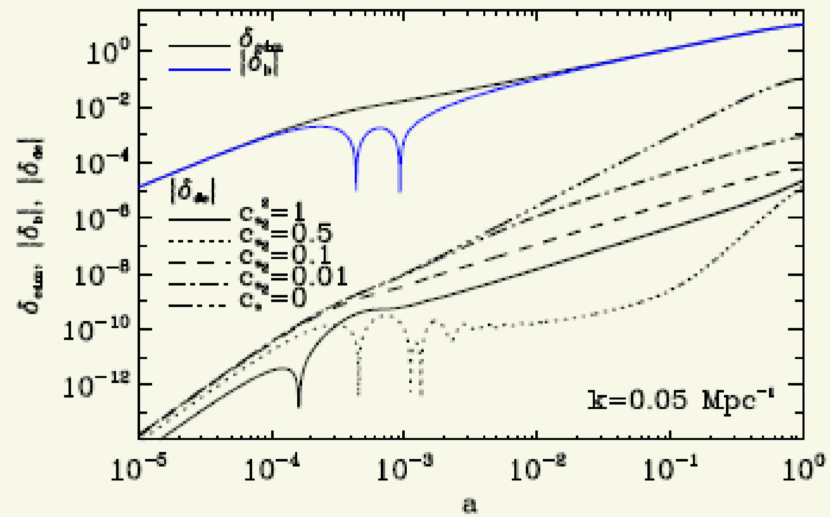
$$\ddot{h} + \frac{\dot{a}}{a}\dot{h} = -8\pi G a^2(\rho_m\delta_m + (1+3w_{de})\rho_{de}\delta_{de})$$

Density perturbations of dark matter, baryon matter and dark energy ($\mathrm{k} = 0.1\mathrm{Mpc}^{-1}$) (CAMB)

$w_{de} = -0.9, c_a^2 = -0.5$



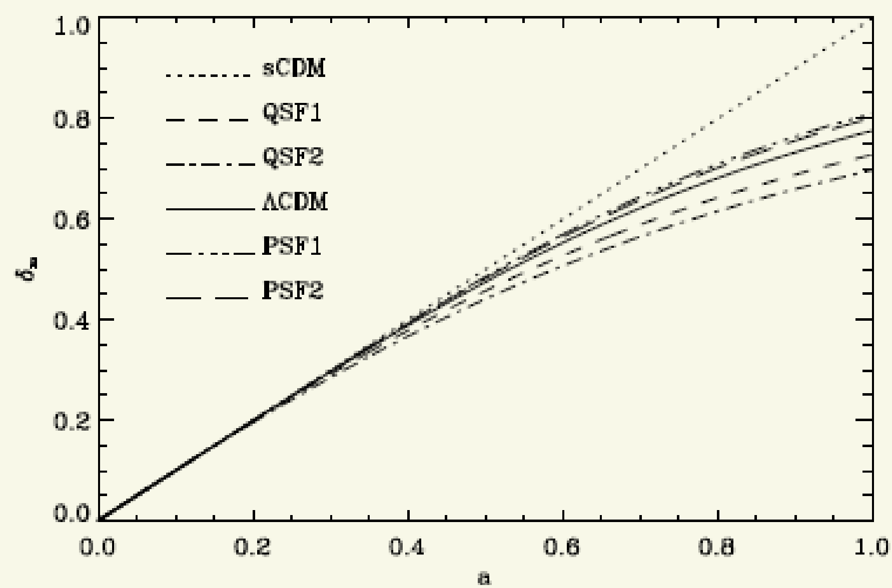
$w_{de} = -1.1, c_a^2 = -1.5$



[Sergijenko & Novosyadlyj, Phys.Rev.D, 92 (2015)]

Evolution of matter density perturbations in the models with different types of DE

The evolution of matter density perturbations from the Dark Ages to the present epoch in sCDM, ΛCDM, QSF+CDM and PSF+CDM models (amplitudes are normalized to 0.1 at $a = 0.1$):

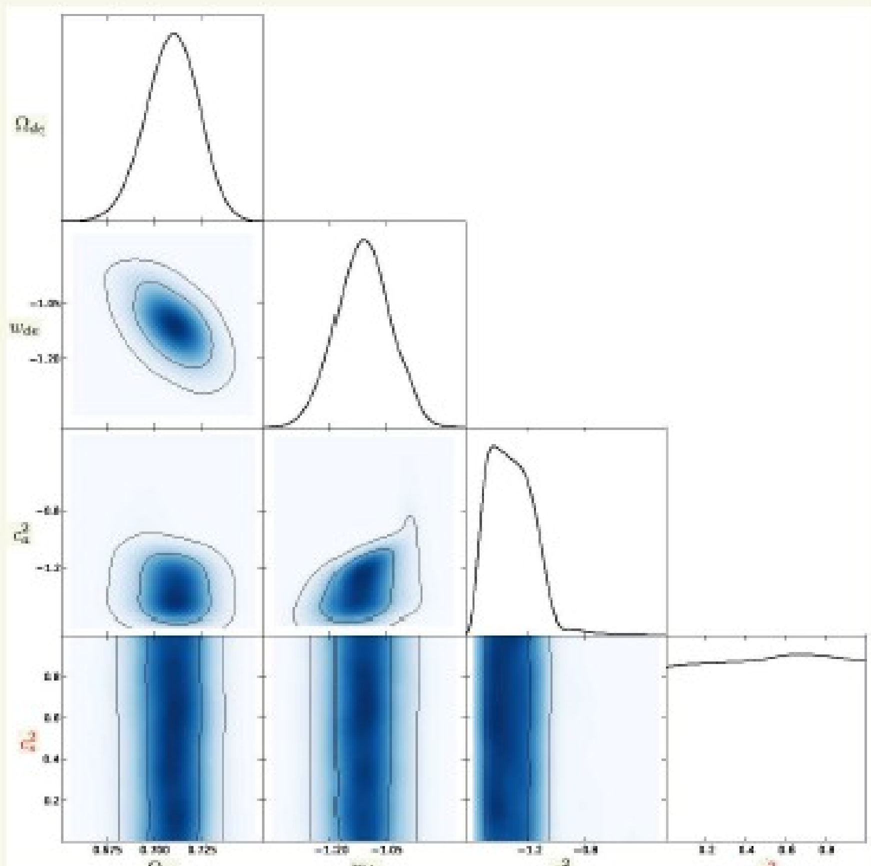


sCDM: $\Omega_m = 1$;
ΛCDM: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$;
QSP1: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$, $w_0 = -0.8$, $c_a^2 = -0.8$;
QSP2: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$, $w_0 = -0.8$, $c_a^2 = -0.5$;
PSP1: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$, $w_0 = -1.2$, $c_a^2 = -1.2$;
PSP2: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$, $w_0 = -1.2$, $c_a^2 = -1.5$;

Current determination of dark energy parameters

Observational data: Planck, WiggleZ, SN Union2.1, H_0

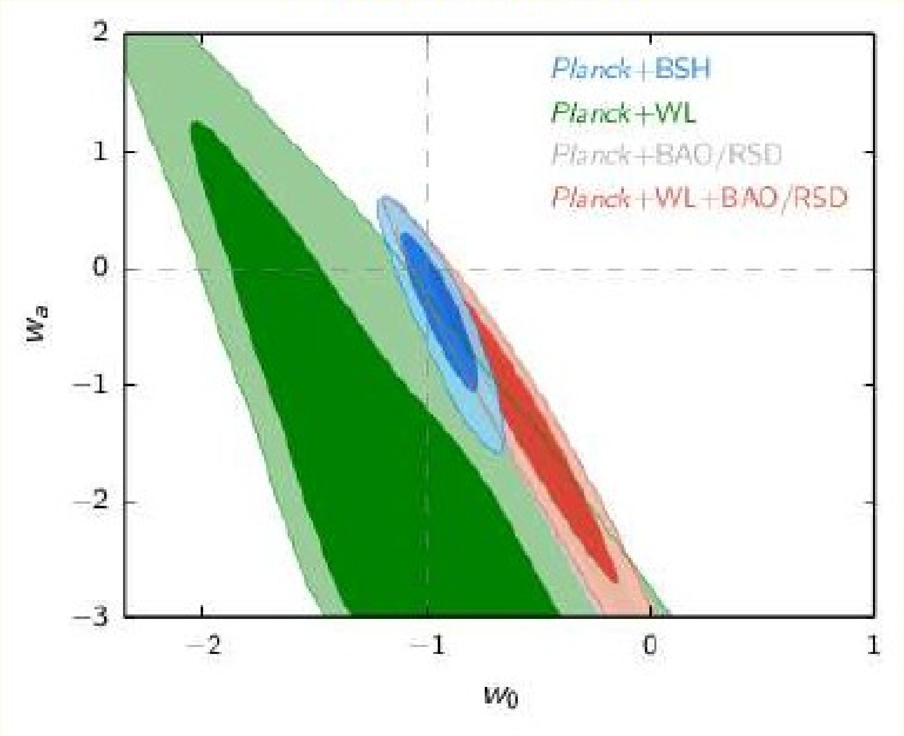
Ω_{de}	w_{de}	c_a^2	c_s^2
$0.71^{+0.03}_{-0.03}$	$-1.11^{+0.14}_{-0.14}$	$-1.32^{+0.25}_{-0.25}$	



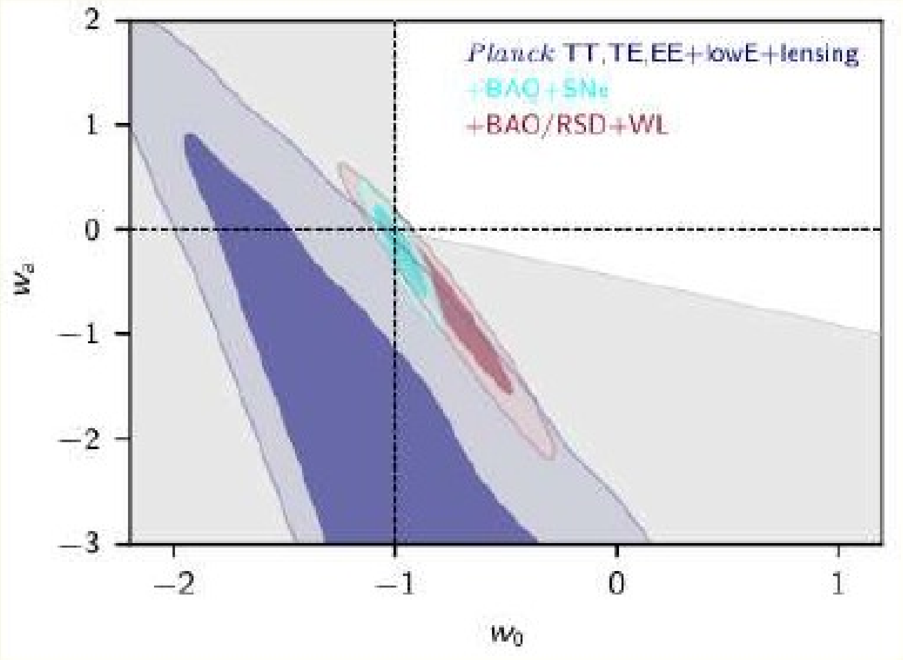
[Sergijenko & Novosyadlyj, Phys.Rev.D, 92 (2015)]

Planck 2018 + other data

Planck 2016



Planck 2018



$$\Omega_{de} = 0.689 \pm 0.006, \quad w_0 = -0.961 \pm 0.077, \quad w_a = -0.28^{+0.31}_{-0.27};$$

$$w_0 = -1.028 \pm 0.032, \quad w_a = 0.$$

Planck Collaboration, Planck 2018 results.VI., arXiv:1807.06209 (2018)

Conclusions I

- Observational data prefer the cosmological model with DE density domination at current epoch: $\Omega_{de} = 0.7 \pm 0.02$.
  The model without DE ($\Omega_{de} = 0$) is excluded at $> 50\sigma$ C.L. !

- Observational data related with cosmological scales (Planck results 2015) give strong constraints on the density of dark energy in the early Universe: $\Omega_{EDE} < 0.0071$.

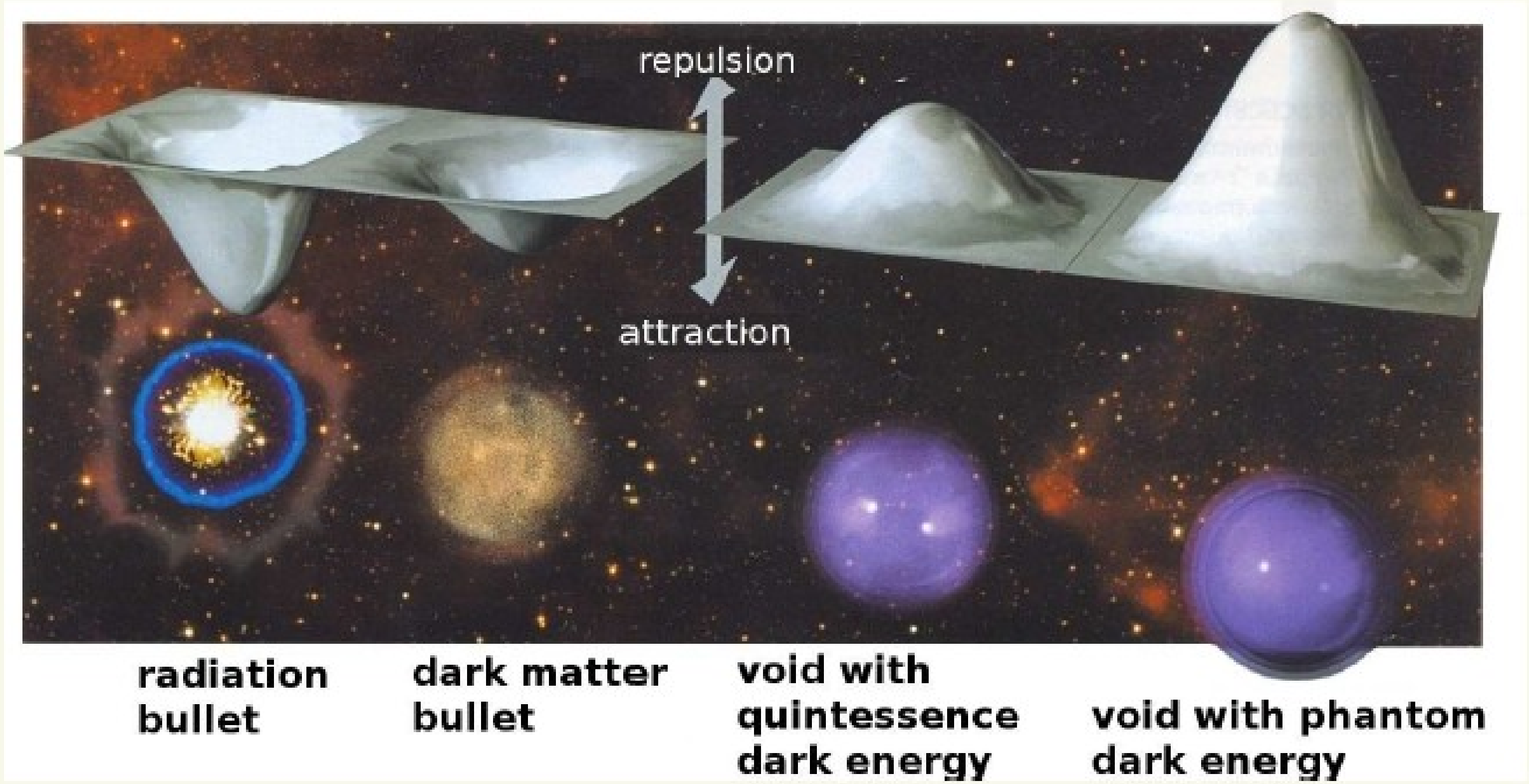
- Observational data related with cosmological scales do not distinguish the DE type: $w_0 = -1.0 \pm 0.15$.

- Currently available observational data related with cosmological scales give no possibility to constrain c_s^2 !

Gravitational potential wells and humps from the different overdensity (clusters) and underdensity (voids) bullets

The ratio of densities of dark energy and matter: $\frac{\rho_{de}}{\rho_m} = \frac{1+\delta_{de}}{1+\delta_m}\frac{\Omega_{de}}{\Omega_m}$,

Universe: $\frac{\rho_{de}}{\rho_m} \approx 2.3$, Halos: $\frac{\rho_{de}}{\rho_m} \ll 1$, Voids: $\frac{\rho_{de}}{\rho_m} \approx 5 - 20$



Source of gravitational field: $c^2\rho + 3p = c^2\rho(1 + 3w)$;
Inertial mass: $c^2\rho + p = c^2\rho(1 + w)$

Spherical perturbations of metric and densities and velocities of matter (m), dark energy (de) and relativistic component (rel)

Metric:

$$ds^2 = e^{\nu(t,r)}dt^2 - a^2(t)e^{\mu(t,r)}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)];$$

Density and pressure:

$$\varepsilon_m(t,r) = \bar{\varepsilon}_m(t)(1+\delta_m(t,r)), \quad p_m(t,r) = 0,$$

$$\varepsilon_{de}(t,r) = \bar{\varepsilon}_{de}(t)(1+\delta_{de}(t,r)), \quad p_{de}(t,r) = w\bar{\varepsilon}_{de} + \delta p_{de}(t,r),$$

$$\delta p_{de}(t,r) = c_s^2\bar{\varepsilon}_{de}\delta_{de}(t,r) - 3\bar{\varepsilon}_{de}aH(1+w)(c_s^2 - w)\int v_{de}(t,r)dr,$$

$$\varepsilon_{rel}(t,r) = \bar{\varepsilon}_{rel}(t)(1+\delta_{rel}(t,r)), \quad p_{rel}(t,r) = \frac{1}{3}\bar{\varepsilon}_{rel}(t)(1+\delta_{rel}(t,r));$$

Velocity:

$$u^i_N(t,r) = \left\{\frac{e^{-\nu/2}}{\sqrt{1-v_N^2}},\ \frac{a^{-1}e^{-\mu/2}v_N}{\sqrt{1-v_N^2}},\ 0,\ 0\right\}, \quad N = m,\ de,\ rel$$

Equations of conservation of energy-momentum $T^k_{i\,;k} = 0$ for dark matter and dark energy:

$$\dot{\delta}_m - \frac{3}{2}(1+\delta_m)\dot{\nu} + \frac{1+\delta_m}{a^2 H}\left(v'_m + \frac{2}{r}v_m\right) + \frac{\delta'_m v_m}{a^2 H} = 0, \tag{2}$$

$$\dot{v}_m + \frac{v_m}{a} + \frac{\nu'}{2a^2 H} + \frac{2v_m}{a^2 H}\left(v'_m + \frac{v_m}{r}\right) + \frac{\dot{\delta}_m v_m}{1+\delta_m} = 0, \tag{3}$$

$$\dot{\delta}_{de} + \frac{3}{a}(c_s^2 - w)\delta_{de} + (1+w)\left[\frac{v'_{de}}{a^2 H} + \frac{2v_{de}}{a^2 H r} - 9H(c_s^2 - w)\int v_{de}dr - \frac{3}{2}\dot{\nu}\right]$$
$$+(1+c_s^2)\left[\frac{\delta'_{de}v_{de}}{a^2 H} + \frac{\delta_{de}}{a^2 H}\left(v'_{de} + \frac{2}{r}v_{de}\right) - \frac{3}{2}\delta_{de}\dot{\nu}\right] = 0, \tag{4}$$

$$\dot{v}_{de} + (1-3c_s^2)\frac{v_{de}}{a} + \frac{c_s^2\delta'_{de}}{a^2 H(1+w)} + \left(1 + \frac{1+c_s^2}{1+w}\delta_{de}\right)\frac{2v_{de}}{a^2 H}\left(v'_{de} + \frac{v_{de}}{r}\right) +$$
$$\frac{\nu'}{2a^2 H} + \frac{1+c_s^2}{1+w}\left[\dot{\delta}_{de}v_{de} + \delta_{de}\dot{v}_{de} + (1-3w)\frac{\delta_{de}}{a}v_{de} + \frac{\nu'\delta_{de}}{2a^2 H}\right] = 0, \tag{5}$$

Initial conditions: amplitudes

We define the initial conditions in the early Universe, when $\rho_{\rm r} \gg \rho_m \gg \rho_{de}$, and physical size of the perturbation $a\lambda \gg ct$.
In that time the perturbations are linear ($\delta,\ v,\ \nu \ll 1$), so without loss of generality the solution can be presented in the form of separated variables:

$$\nu(a,r) = \tilde{\nu}(a)f(r), \quad \delta_N(a,r) = \tilde{\delta}_N(a)f(r), \quad v_N(a,r) = \tilde{v}_N(a)f'(r),$$

where $f(0) = 1$ and $f'(r) \propto r$ near the center $r = 0$.
The relations for amplitudes of growth mode of perturbations in the superhorizon asymptotic at the radiation-dominated epoch:

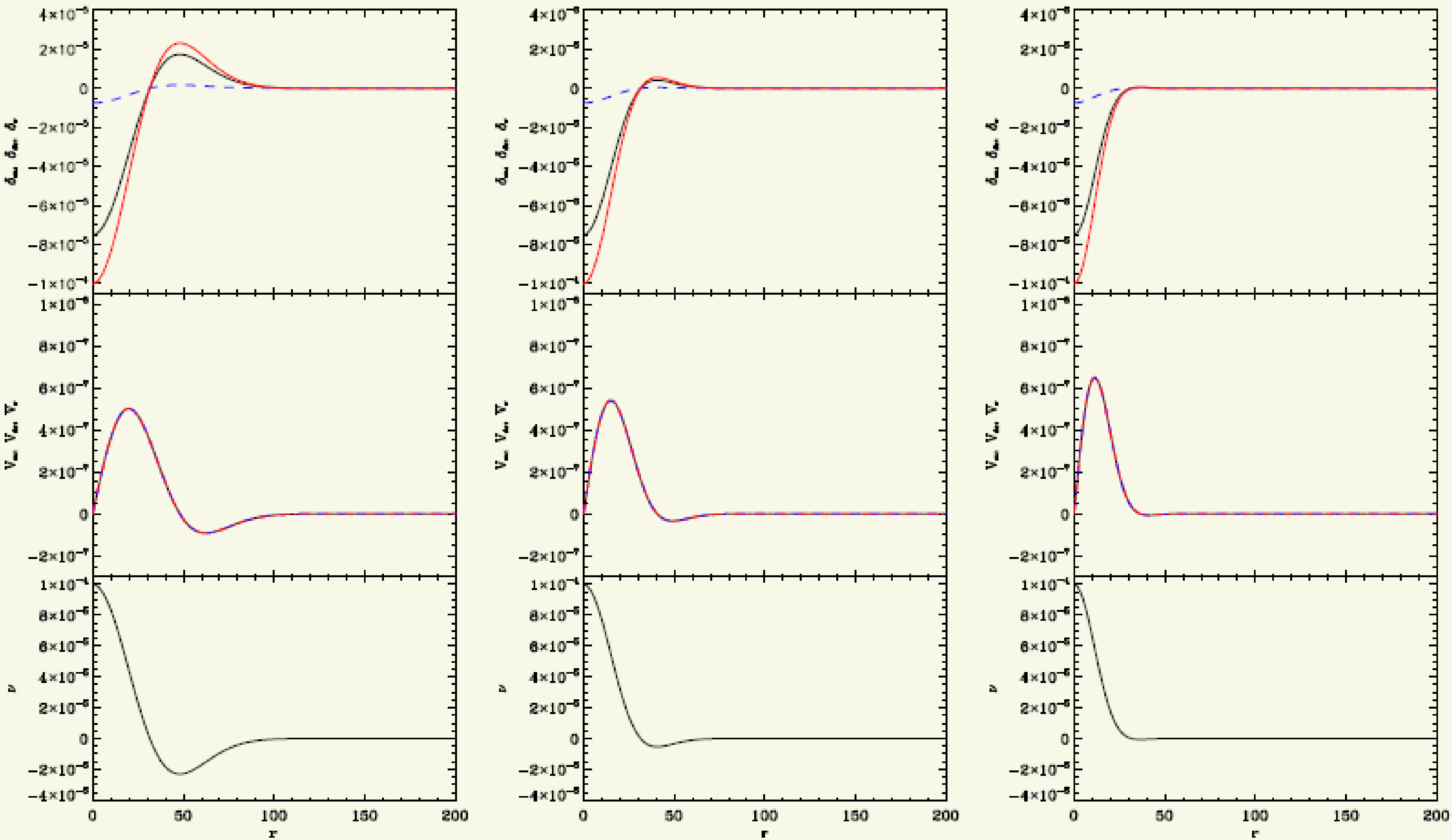
$$\tilde{\delta}_{rel}^{init} = C, \quad \tilde{v}_{rel}^{init} = \frac{C}{4a_{init}H(a_{init})}, \quad \tilde{\nu}^{init} = -C,$$

$$\tilde{\delta}_{m}^{init} = \frac{3}{4}C, \quad \tilde{v}_{m}^{init} = \frac{C}{4a_{init}H(a_{init})},$$

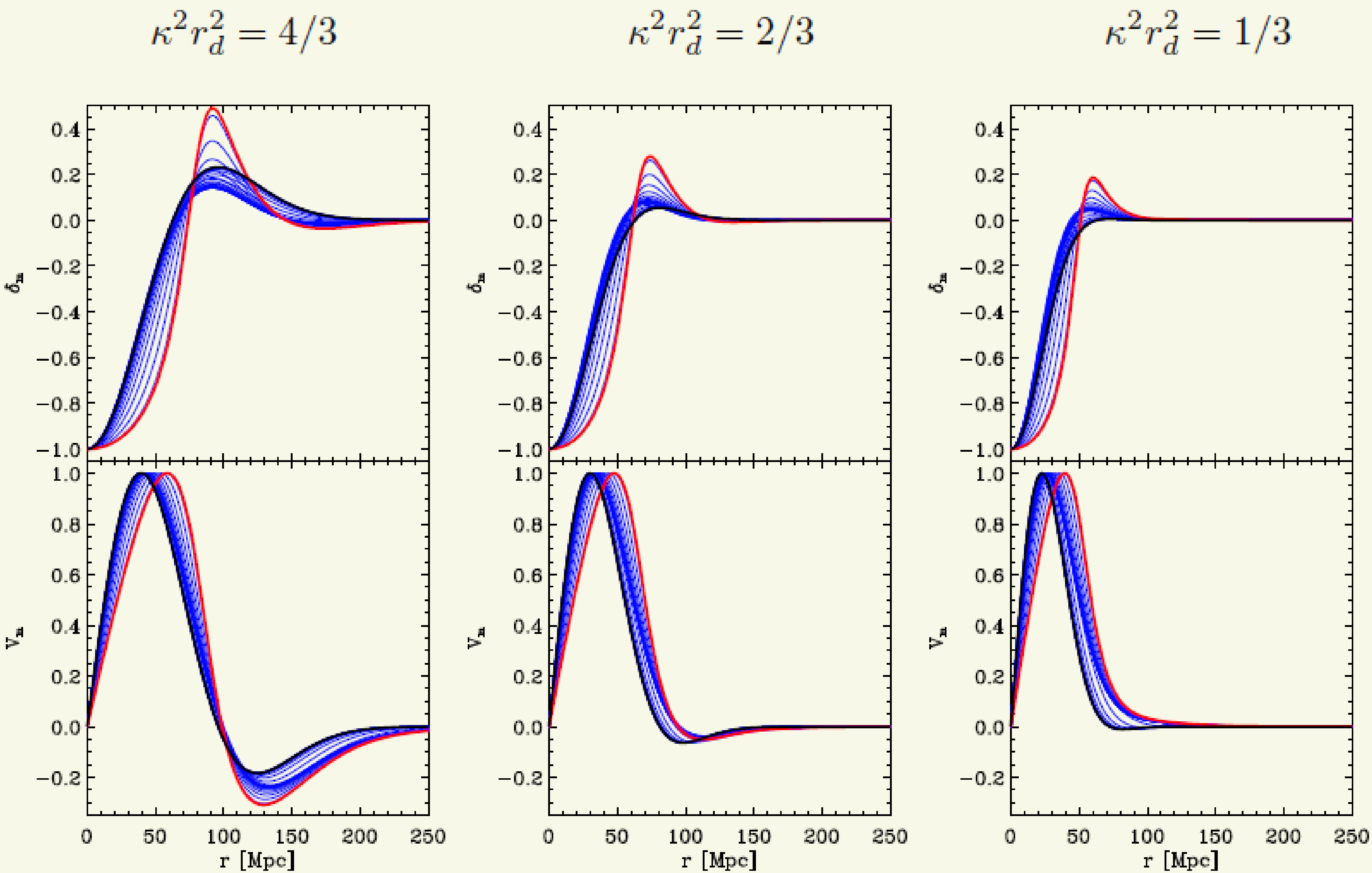
$$\tilde{\delta}_{de}^{init} = \frac{3}{4}(1+w)C, \quad \tilde{v}_{de}^{init} = \frac{C}{4a_{init}H(a_{init})}.$$

Initial profiles: $\delta^{init}(r) \propto (1-\kappa r^2)e^{-r^2/r_d^2}$, $\kappa \equiv r_s^{-2} = (\pi/k)^{-2}$, $r_s = 62.8$ Mpc, $C = -1 \cdot 10^{-4}$, $a_{init} = 10^{-6}$ ($z_{init} \equiv a_{init}^{-1} - 1 \approx 10^6$)

$\kappa^2 r_d^2 = 4/3$ $\quad$ $\kappa^2 r_d^2 = 2/3$ $\quad$ $\kappa^2 r_d^2 = 1/3$

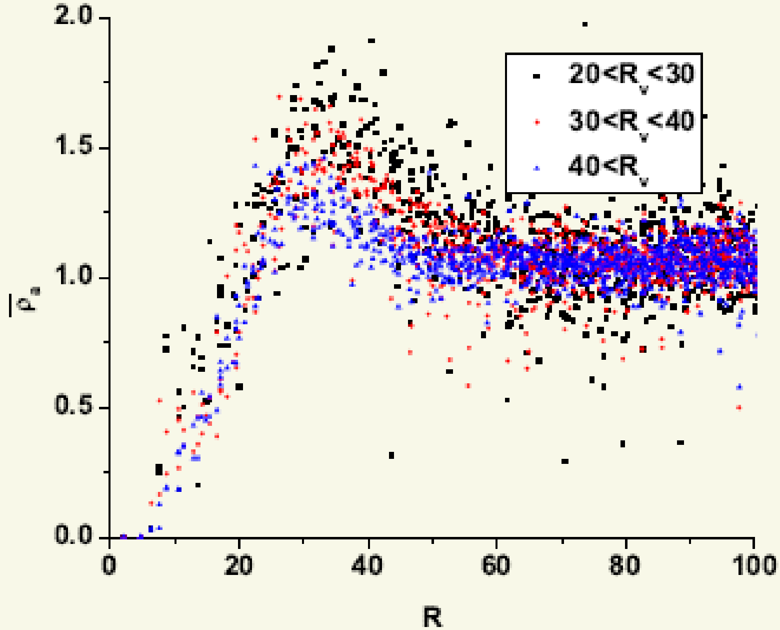


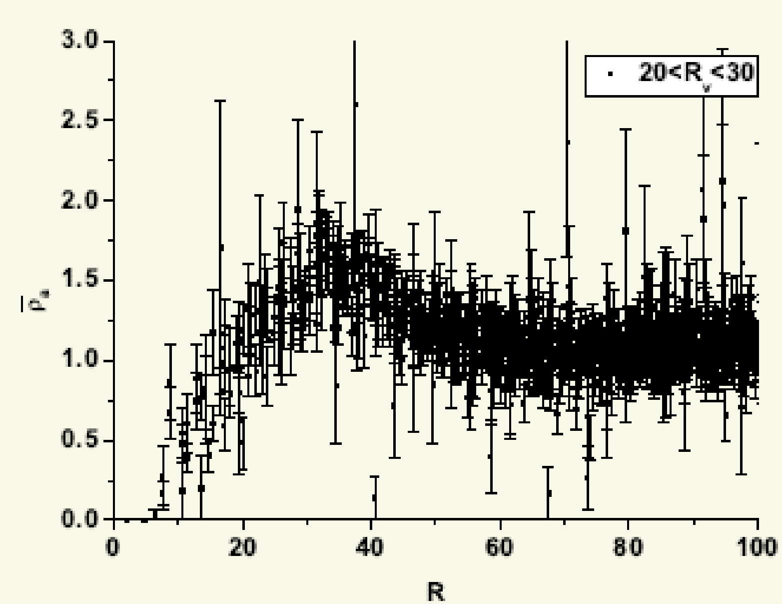
Evolution of matter density and velocity profiles ($\Omega_{de} = 0.7$, $w = -0.9$, $c_s^2 = 1$, $r_s = 62.8$ Mpc, $C_k = -2 \cdot 10^{-4}$).



Profiles of real voids

Public Cosmic Void catalog, created on the base of SDSS DR7-10, contains 2325 voids. [Sutter et al., ApJ, 761, 44 (2012); MNRAS, 442, 3127 (2014); MNRAS, 443, 2983 (2014)]

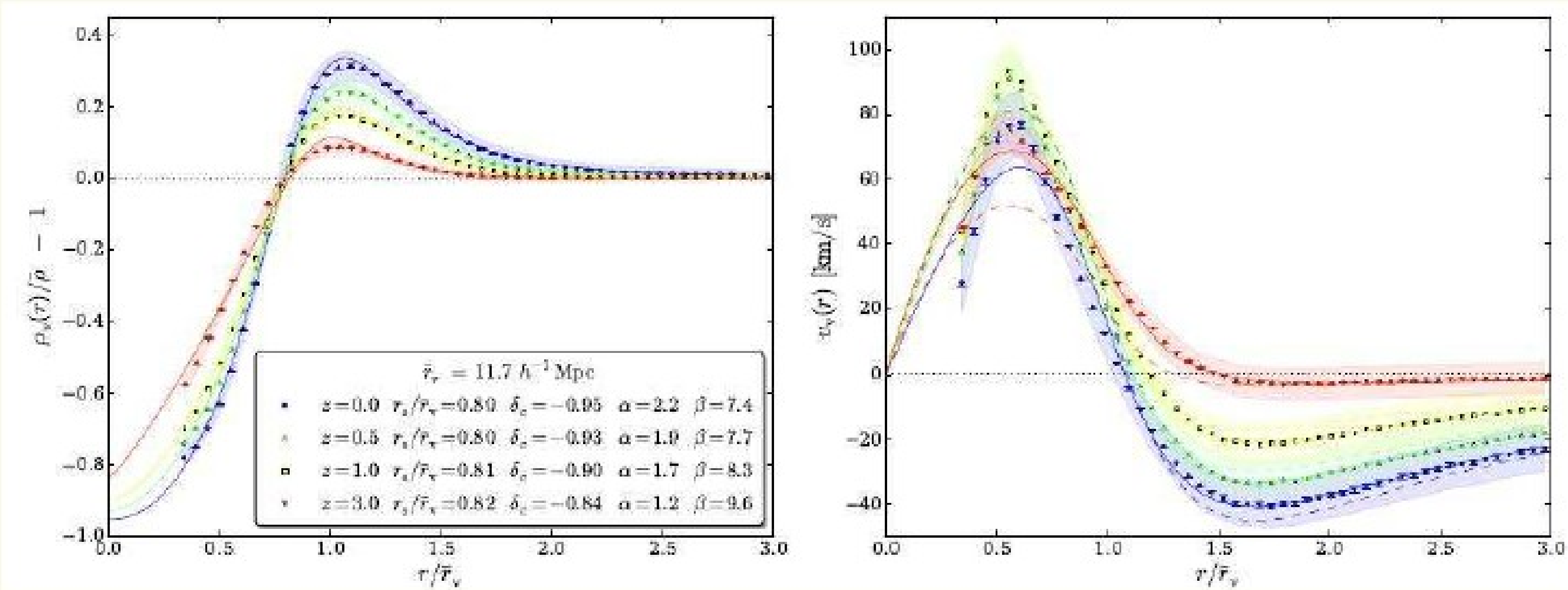




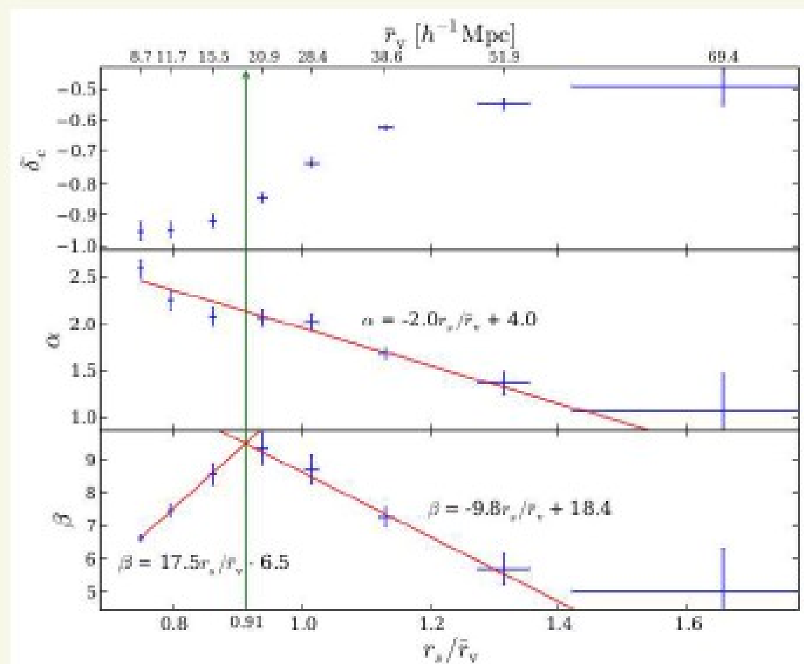
696 voids at $z = 0.5 - 0.6$

[*V*oid alignment and density profile applied to measuring cosmological parameters, Dai D.-C., MNRAS, 454, 3590 (2015)]

Universal Density Profile for Cosmic Voids, Hamaus N. et al., Phys.Rev.Lett., 112, id. 251302 (2014)

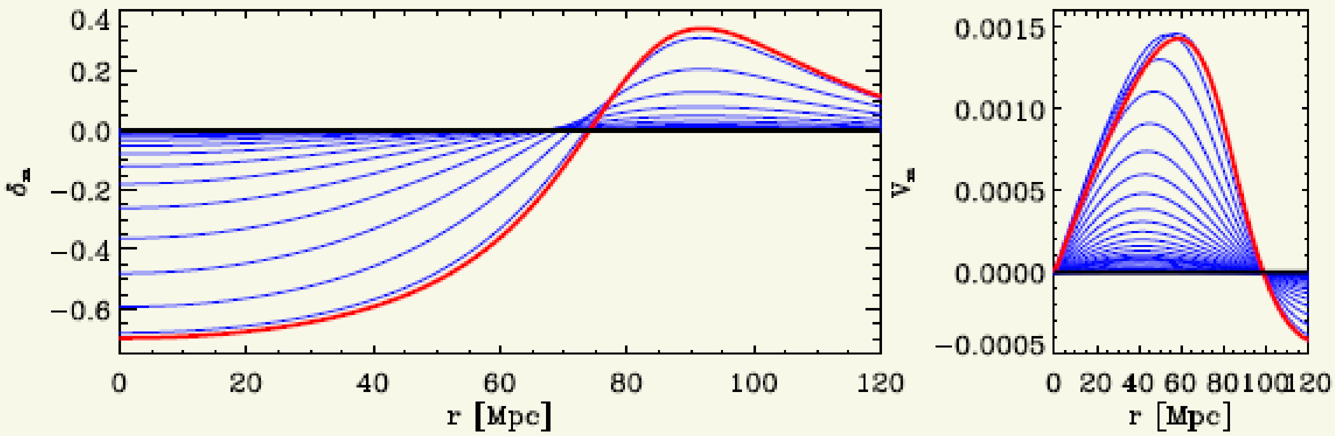


$$\delta(r) = \delta_0 \frac{1-(r/r_s)^{\alpha}}{1+(r/r_v)^{\beta}},$$



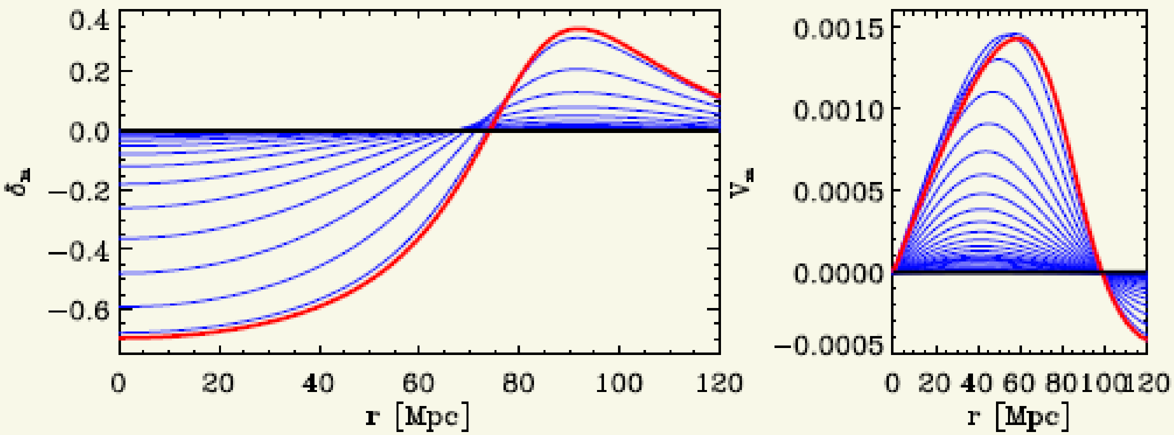
Sensitivity of void parameters to DE c_s^2

$\frac{\delta_m(c_s^2=0)-\delta_m(c_s^2=1)}{\delta_m(c_s^2=1)}$ void	$\frac{\delta_m(c_s^2=0)-\delta_m(c_s^2=1)}{\delta_m(c_s^2=1)}$ overdensity shell	$\frac{v_m(c_s^2=0)-v_m(c_s^2=1)}{v_m(c_s^2=1)}$ void
0.1-0.2 %	0.4-0.5 %	1.5-1.6 %



Sensitivity of void parameters to dark energy EoS parameter w

w	$\frac{\delta_m(w)-\delta_m(-1)}{\delta_m(-1)}$ void	$\frac{\delta_m(w)-\delta_m(-1)}{\delta_m(-1)}$ overdensity shell	$\frac{v_m(w)-v_m(-1)}{v_m(-1)}$ void
-0.8	2-4 %	7-9 %	6-7 %
-0.9	1-2 %	3-4 %	$\sim$3 %
-1.1	$\sim$1 %	3-4 %	$\sim$3 %
-1.2	$\sim$2 %	5-6 %	$\sim$4 %



Equation for amplitudes at center of spherical halo:

$$\dot{\tilde{\delta}}_m - \frac{3}{2}(1+\tilde{\delta}_m)\dot{\tilde{\nu}} - \frac{1+\tilde{\delta}_m}{a^2H}k^2\tilde{v}_m = 0, \tag{8}$$

$$\dot{\tilde{v}}_m + \frac{\tilde{v}_m}{a} + \frac{\tilde{\nu}\delta_m}{2a^2H} + \frac{\dot{\tilde{\delta}}_m\tilde{v}_m}{1+\tilde{\delta}_m} - \frac{4k^2\tilde{v}_m^2}{3a^2H} = 0, \tag{9}$$

$$\dot{\tilde{\delta}}_{de} + \frac{3}{a}(c_s^2 - w)\tilde{\delta}_{de} - (1+w)\left[\frac{k^2\tilde{v}_{de}}{a^2H} + 9H(c_s^2 - w)\tilde{v}_{de} + \frac{3}{2}\dot{\tilde{\nu}}\right] - (1+c_s^2)\left[\frac{k^2\tilde{v}_{de}}{a^2H} + \frac{3}{2}\dot{\tilde{\nu}}\right]\tilde{\delta}_{de} = 0, \tag{10}$$

$$\dot{\tilde{v}}_{de} + (1-3c_s^2)\frac{\tilde{v}_{de}}{a} + \frac{c_s^2\tilde{\delta}_{de}}{a^2H(1+w)} + \frac{\tilde{\nu}}{2a^2H} - \frac{4k^2\tilde{v}_{de}^2}{3a^2H} + \frac{1+c_s^2}{1+w}\left[\dot{\tilde{\delta}}_{de}\tilde{v}_{de} + \tilde{\delta}_{de}\dot{\tilde{v}}_{de} + (1-3w)\frac{\tilde{\delta}_{de}}{a}\tilde{v}_{de} + \frac{\tilde{\nu}\tilde{\delta}_{de}}{2a^2H}\right] = 0, \tag{11}$$

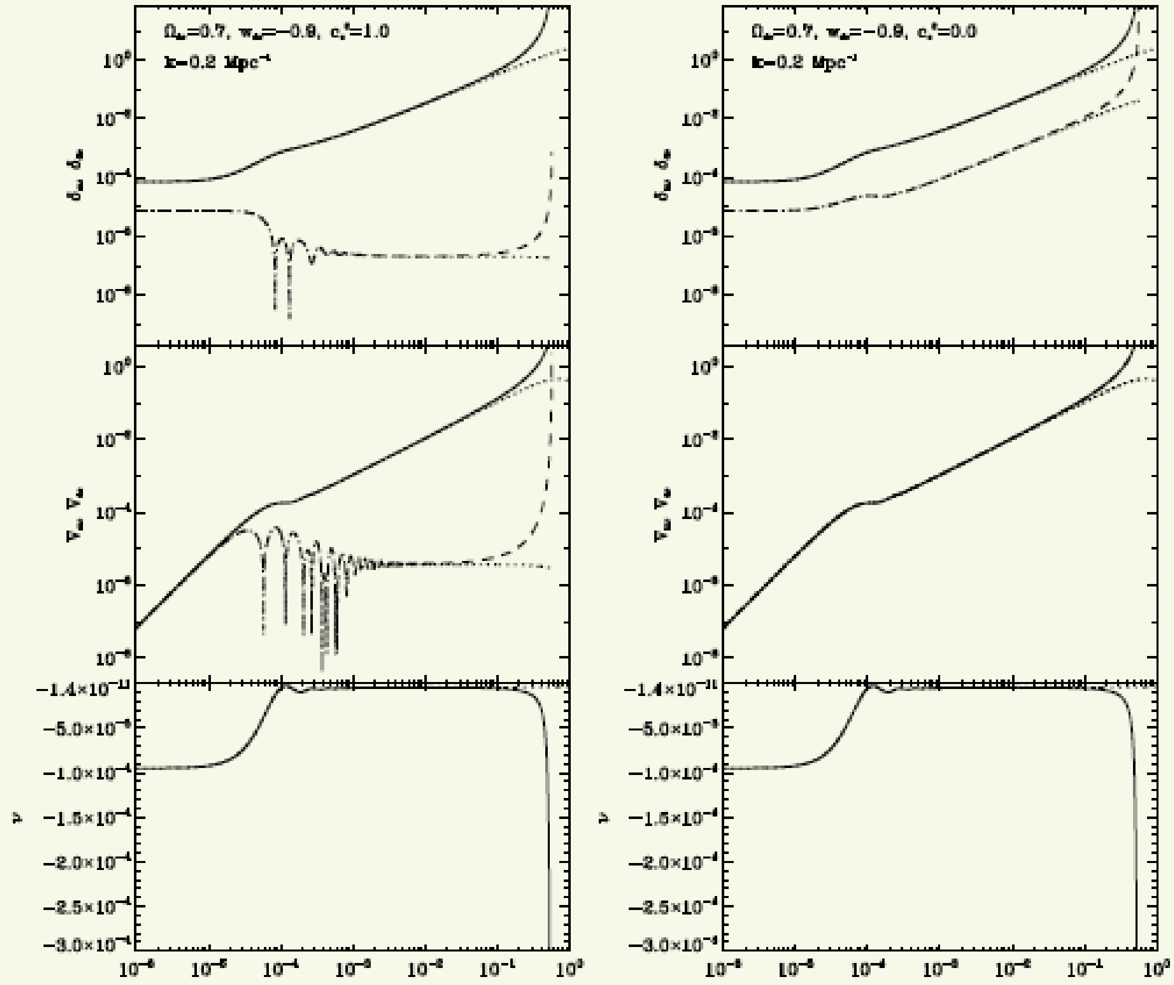
$$\dot{\tilde{\delta}}_{rel} - 2(1+\tilde{\delta}_{rel})\dot{\tilde{\nu}} - \frac{4}{3}\frac{1+\tilde{\delta}_{rel}}{a^2H}k^2\tilde{v}_{rel} = 0, \tag{12}$$

$$\dot{\tilde{v}}_{rel} + \frac{\tilde{\nu}}{2a^2H} + \frac{\tilde{\delta}_{rel}}{4a^2H(1+\tilde{\delta}_{rel})} + \frac{\dot{\tilde{\delta}}_{rel}\tilde{v}_{rel}}{1+\tilde{\delta}_{rel}} = 0, \tag{13}$$

$$\dot{\tilde{\nu}} + \left(1 + \frac{k^2}{3a^2H^2}\right)\frac{\tilde{\nu}}{a} = -\frac{H_0^2}{aH^2}\left(\Omega_m a^{-3}\tilde{\delta}_m + \Omega_{de}a^{-3(1+w)}\tilde{\delta}_{de}\right). \tag{14}$$

Publicly available code: http://194.44.198.6/∼novos/dedmhalo.tar.gz

Evolution of amplitudes at the center of spherical halo with $k = 0.2$ Mpc^{-1}, which is collapsing now



$$\upsilon_N \Longrightarrow V_N = \frac{\upsilon_N}{\upsilon_H} = -\frac{1}{3}\frac{k^2 \tilde{v}_m}{aH}, \quad \upsilon_H = aHr$$

Turn-around of matter $a_{ta} \approx 0.35, \quad (z_{ta} \approx 1.9)$.

Dynamic of dark energy in the halo, which has been formed at $z \approx 10$

Phenomenological approach for halo virialization:

1. To slow down the fall of matter we introduce into eq. of motion of matter the artificial bulk viscosity:

$$F_v = \sigma_v \left(\frac{\tilde{\delta}_m}{\Delta_v} \right)^2,$$

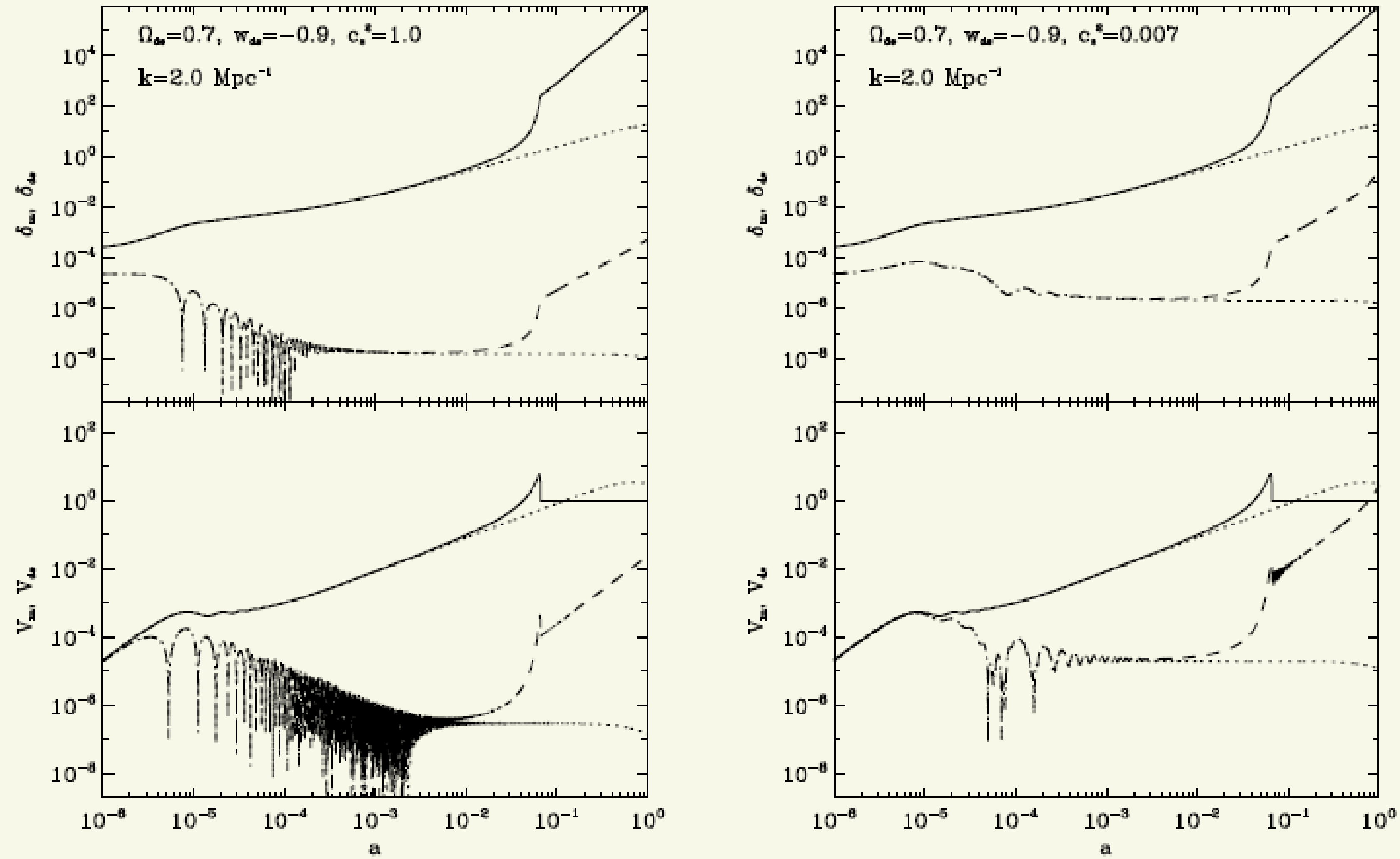
$\Delta_v = 100 - 178$ *[Kulinich, Novosyadlyj & Apunevych, Phys.Rev.D, **88**, (2013)]*

2. to keep the density of matter at the given constant value $\rho_v = \Delta_v \rho_{cr}(z_v)$:

the equation for $\tilde{\delta}_m \Longrightarrow \dot{\tilde{\delta}}_m - 3\Delta_v \frac{a^2}{a_v^3} = 0$,

the equation for $\tilde{v}_m \Longrightarrow \dot{\tilde{v}}_m - 3\left(H + a\frac{dH}{da}\right) = 0$.

Evolution of amplitudes at the center of spherical halo, which has been formed $z = 14$ ($k = 2$ Mpc^{-1})



Turn-around of matter: $a_{ta} \approx 0.042, \quad (z_{ta} \approx 23)$

Virialization of spherical halo: $a_v \approx 0.067, \quad (z_v \approx 14)$

Conclusions II

- Dynamical properties of dark energy in the voids and halo depend on the value of effective sound speed.
- Dark energy with $c_s = 1$ is practically unperturbed in the voids and halo.
- Dark energy with $c_s = 0$ moves together with dark matter at all phases of evolution.
- The amplitudes of matter density in voids and in their overdensity shells as well as peculiar velocity depend on c_s of dark energy at the level of few percents.
- The amplitudes of matter density in voids and in their overdensity shells as well as the peculiar velocity are sensitive to the value of EoS parameter of dark energy w_{de} at the level of few percents.

Dynamical dark energy in the vicinity of stars: static solution

The space-time metric in spherically symmetric coordinates:

$$ds^2 = e^{\nu(r)} d\tau^2 - e^{\lambda(r)} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

We consider the dynamics of scalar field dark energy in the gravitational field of homogeneous spherical object with radius R:

$$ds^2 = (1 - r_g/r)\, d\tau^2 - \frac{dr^2}{1 - r_g/r} - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

for $r \geq R$ and

$$ds^2 = \frac{(1-\alpha)^{3/2}}{(1 - \alpha r^2/R^2)^{1/2}} d\tau^2 - \frac{dr^2}{1 - \alpha r^2/R^2} - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

for $r \leq R$.

Here

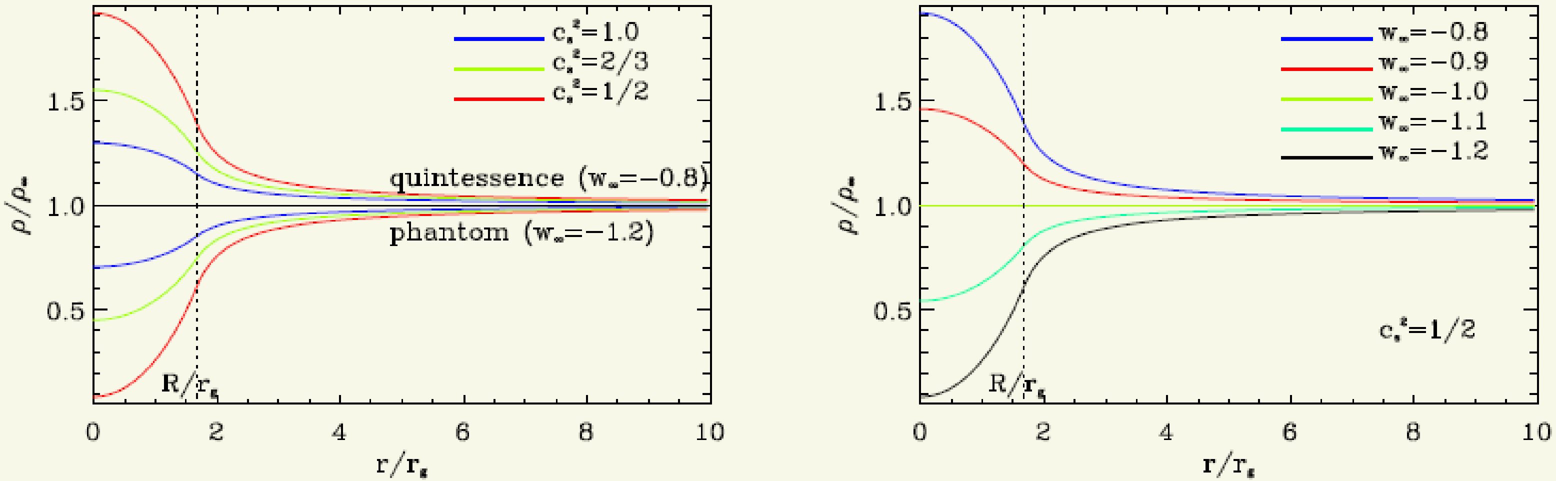
$$r_g = \frac{2GM}{c^2}, \quad \alpha \equiv \frac{r_g}{R}.$$

Dynamical dark energy in the vicinity of stars: static solution

Conservation law for probe dark energy in the stationary gravitational field:

$$w_{de} = c_s^2 - (c_s^2 - w_\infty)\frac{\rho_\infty}{\rho}.$$

$$T_{i;k}^{k\,(de)} = 0 \quad \rightarrow \quad \frac{dp}{dr} + \frac{1}{2}(\rho + p)\frac{d\nu}{dr} = 0.$$



$$\rho(r) = \rho_\infty\left(\frac{c_s^2 - w_\infty}{1 + c_s^2} + \frac{1 + w_\infty}{1 + c_s^2}\left[e^{\nu(r)}\right]^{-\frac{1+c_s^2}{2c_s^2}}\right),$$

where $e^{\nu(r)} = 1 - r_g/r$ outside the spherical object and $e^{\nu(r)} = (1-\alpha)^{3/2}/(1-\alpha r^2/R^2)^{1/2}$ inside it.

Dynamical dark energy in the centers of astrophysical objects

Object	Mass M	r_g	Radius R	$(\rho(0)-\rho_\infty)/\rho_\infty$
	kg	m	m	$1-w_\infty=\pm 0.2$
Earth like planet	$6\cdot 10^{24}$	$1\cdot 10^{-2}$	$6\cdot 10^{6}$	$\pm 2.5\cdot 10^{-10}$
Sun like star	$2\cdot 10^{30}$	$3\cdot 10^{3}$	$7\cdot 10^{8}$	$\pm 6.4\cdot 10^{-7}$
White dwarf	$2\cdot 10^{30}$	$3\cdot 10^{3}$	$6\cdot 10^{6}$	$\pm 7.5\cdot 10^{-5}$
Neutron star	$4\cdot 10^{30}$	$6\cdot 10^{3}$	$1\cdot 10^{4}$	± 0.3
Galaxy	$3\cdot 10^{42}$	$5\cdot 10^{15}$	$6\cdot 10^{20}$	$\pm 1.3\cdot 10^{-6}$
Cluster of galaxy	$2\cdot 10^{44}$	$3\cdot 10^{17}$	$5\cdot 10^{22}$	$\pm 9\cdot 10^{-7}$

Table 1: The estimations of deflection of the densities of quintessence and phantom dark energy in the centers of different gravitationally bound objects, $(\rho(0)-\rho_\infty)/\rho_\infty$. In the evaluation we take $c_s^2=1$ along with $w_\infty=-0.8$ for quintessential dark energy and $w_\infty=-1.2$ for phantom one.

[Novosyadlyj, Kulinich & Tsizh, arXiv:1404.0276; ; Phys.Rev D 90, 063004 (2014)]

Stationary accretion of dark energy on black hole

Presentation of 4-velocity $u^i \equiv dx^i/ds$ of DE via its 3-velocity $v(r) \equiv d\tilde{r}/d\tilde{\tau}$:

$$u_i = \left\{ \frac{e^{\nu(r)/2}}{\sqrt{1-v^2}}, -\frac{v e^{\lambda(r)/2}}{\sqrt{1-v^2}}, 0, 0 \right\}, \quad u^i = \left\{ \frac{e^{-\nu(r)/2}}{\sqrt{1-v^2}}, \frac{v e^{-\lambda(r)/2}}{\sqrt{1-v^2}}, 0, 0 \right\}.$$

Motion equations for DE:

$$T^{i(de)}_{0\,;i} = 0 \quad \rightarrow \quad (1+w)\rho r^2 \frac{v}{1-v^2} e^{\nu} = C$$

$$T^{i(de)}_{1\,;i} = 0 \quad \rightarrow \quad \frac{1}{1+w}\frac{d\ln\rho}{dr} + \frac{1}{1-v^2}\frac{d\ln v}{dr} - \frac{1}{2}\frac{d\lambda}{dr} + \frac{2}{r} = 0.$$

Stationary accretion of dark energy on black hole

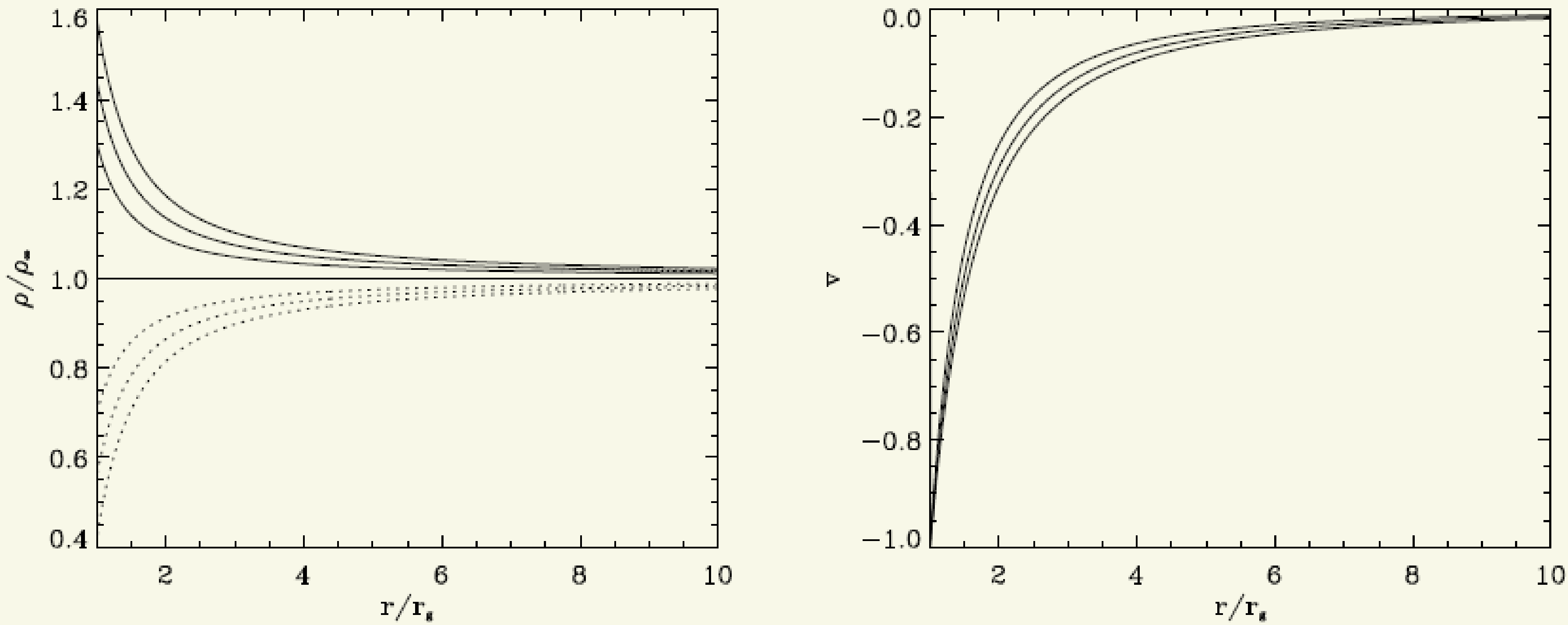


Figure 1: The dependence of dark energy and radial component of 3-velocity on distance to the center of black hole for models of dark energy with $w_\infty = -0.8$ (solid lines) and $w_\infty = -1.2$ (dotted lines) and three values of square of effective sound speed c_s^2 =1, 2/3, 1/2.

[Novosyadlyj, Kulinich & Tsizh, arXiv:1404.0276; Phys.Rev D 90, 063004 (2014)]

Estimation of lower limit for c_s of DE

EPM2011: 677 000 positional observations of the planets and spacecrafts give $\Delta M \leq 5 \cdot 10^{-9} M_{\odot}$ ($r \leq \cdot 10^9$ km) [Pitjev & Pitjeva, Astronomy Letters, 39, 141 (2013)]

$$M_{TE}(r_1) = 4\pi\rho_\infty \left[(c_s^2 - w_\infty) \left(\frac{1+3c_s^2}{1+c_s^2} - 3 \right) r_1^3/3 + \right.$$

$$\left. + \frac{(1+w_\infty)(1+3c_s^2)}{1+c_s^2} \left(\int_0^{R_\odot} \left[\frac{(1-\frac{R_g}{R_\odot})^{3/2}}{1-\frac{r^2 R_g}{R_\odot^3}} \right]^{-\frac{1+c_s^2}{2c_s^2}} r^2 dr + \int_{R_\odot}^{r_1} (1-\frac{R_g}{r})^{-\frac{1+c_s^2}{2c_s^2}} r^2 dr \right) \right]$$

$$c_s > 2 \cdot 10^{-4} \left[\frac{(1+w_\infty)\rho_\infty}{1.2 \cdot 10^{-27}} \right]^\beta, \qquad \beta \approx 0.17$$

[Tsizh, Novosyadlyj & Kulinich, AASP, 5, 51 (2015)]

Conclusions III

- In the static world of galaxies or clusters of galaxies dark energy average density doesn't change with time, $\overline{\rho}_{de}^{(gal,\,cl)} \ll \overline{\rho}_{m}^{(gal,\,cl)}$. DE is gravitationally stable.
- In the gravitational fields of stars and planets the static solution exists and describes the distribution of dark energy density near and inside the stars. Its value differs slightly from the mean one, the magnitude of relative deviation of the density, $\delta_{de} = \rho_{de}(0)/\overline{\rho}_{de} - 1$, has maximum in the center and depends on the depth of the potential well, value of EoS parameter and effective sound speed.
- The solutions for stationary accretion of DE on the Schwarzschild black hole show that the rate of inflow of DE depends on the parameters of dark energy w_∞ and c_s^2.
- The density of phantom dark energy in the potential well of concentration of matter is lower than average one and tends to zero when the size of the object approaches the value of gravitational radius.