PHOTOMETRIC METHOD FOR DETERMINING A PLANETARY SURFACE RELIEF.

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Methods for determining the surface relief. A number of methods for determining a relief of the planet's surface exists. A schlieren method (based on the length of shadows) was one of the first ones used to determine heights of various formations on the Moon [1]. In 1951, a photometric (photoclinometric) method was proposed [2], which makes use of the known dependence of the area brightness on its orientation. A photogrammetric (stereoscopic) method is widely used as well, in particular, in exploration of the Earth's surface. With the advent of the space exploration era, a direct altimetry with the optical or radio radars has become of considerable current use. A possibility appeared to apply a photoclinometric method with the use of radar images, in particular, those ones obtained with the synthetic aperture radars (SAR) [3]. Currently, the interferometric method is being developed, which can be attributed to radio holography [4,5].

Photometric method. Each of these methods has the intrinsic advanced features and weak points. Their analysis might become a subject of a separate investigation. This presentation is dedicated to only one of them, viz, to the photometric method, named also the method of photoclinometry, since in some (simplified) versions its application consists of two stages: creation of the field of inclinations from images, and restoration of the relief from the field of inclinations. The first task of these two ones presents no special problems, but requires, however, a detailed knowledge of photometric properties of the surface under consideration. The second task was investigated for the first time in [2], where the most simple approach to solve it is presented. As it often takes place in solving the inverse problems in physics and astronomy, this problem turns out to be mathematically incorrect in this simplest statement and requires more accurate formulation. Posing of this problem on the basis of the Bayesian statistical approach [6] seems to be most natural, as has been indicated in [7].

Bayesian approach to solving the inverse problems. The Bayesian statistical approach originates from the works by T.Bayes [8], P.S.Laplace [9], K.F.Gauss [10], and A.M.Legandre [11], but it was properly advanced as late as in the middle of the 20-th century [12], and described carefully in [6]. In spite of its importance for practice (for example, in radio- location) and of principal significance in scientific research, especially in experimental physics and observational astronomy, it has not become a current instrument in experimental studies. This caused, in particular, publication of a review [13], which admonishes the reader of its importance. A review [] is dedicated to the role of this approach in image filtration problem and, in particular, in the problem of the relief restoration from images [14].

Posing the problem of the relief restoration on the basis of the Bayesian approach. The problem of determining the relief from the field of inclinations looks in the following way in this approach. Let the field of inclinations t(x,y) be given, which is known with a random error distributed normally, with the same variance over the whole area, and at different points independently. A prior probability density distribution of the sought relief is also known. Let the problem be posed in such a way that every probable relief be a realization of the same stationary Gaussian process known a priori. It is required to find such a realization H(x,y) of this process, which would meet a requirement of the maximal a posterior probability density with given t(x,y). This problem is reduced to minimization of a quadratic functional, and ultimately, to solving a Poison equation for H(x,y), with the t(x,y) divergence in its right side. The boundary condition depends on additional information about the relief. In particular, if the relief is specified at the area border, this is just the boundary condition. In the full absence of additional information about the relief, the minimum of the functional reaches if the normal derivative of H(x,y) equals zero at the border.

Possibility to consider the problems with the data excess and deficit from a single point of view. Such an approach makes it possible both to correctly take into account the redundant information (when many images at different illumination conditions are given), and to estimate the relief under the lack of information (when the initial data are insufficient to determine the relief even in the absence of measurement errors, e.g., when a single image is available). The problem of the relief estimation from a single image occurs rather frequently. There are many dedicated studies [15-17]. A properly posed approach might facilitate its solution significantly [18].

Matching initial images. In determining the relief, initial images are usually not matched in advance, and thus, their matching is needed. The problem is not simple if the images were obtained under different conditions of illumination. One approach may consist in determining the relief from each image separately, with the subsequent matching of the obtained maps of the relief [19]. This method turns out to be rather efficient. More drastic approach consists in solving, at the very beginning, the problem of finding the most probable set of functions H(x,y) and vectors of image shifts τ_i.

Simultaneous determination of the relief and optical parameters of the surface. Everything discussed above is tightly connected with another question. The brightness of the surface element under the given conditions of illumination and observations depends not only on its orientation, but also on its optical characteristics, e.g., on albedo. Such characteristics are not always known in advance. Therefore, for optically inhomogeneous surface areas one should pose the problem of simultaneous determination of the relief and optical parameters (from a sufficient set of initial images obtained under various conditions of illumination), and, if the need arises, with simultaneous superposition of images and removing their blurring caused by the propagation media and imperfectness of facilities. This problem was set and solved in a linear approximation by [20]. Analysis of random errors of this method caused by the presence of spatial noise in initial images, is presented in [21].

Accounting for the altimetry information in determining the relief from the field of inclinations. As a rule, altimetry gives heights of a surface at separate points, which form a rather sparse mesh, and thus ranks below the clinometry in resolution. However, the accuracy of altimetry can be much higher than that of the photometric method, therefore, it is desirable that the data of altimetry be taken into account in determining the relief with clinometry. It is easy to do considering definition of heights in these points as an additional boundary condition and solving the Poisson equation for $H(x,y)$ with this strengthened boundary condition.

Computational aspects of the relief determination from the field of inclinations. In spite of the power of the current computer technique, one should choose the algorithms that would be more economic regarding the required memory and computer time. A net method is a natural way to numerically solve the Poisson equation. It requires a computer time that is proportional to the squared number of the net nodes. Therefore, a search for ways to accelerate the process is of interest. In doing so, a properly applied fast Fourier transform could be helpful.

Conclusion. A photometric (photoclinimetric) method on basis of the Bayesian statistical approach is substantially developed in [7,20,3,21,14,18]. However, new questions will be arising in its practical application, and thus investigations in this field should not be regarded as accomplished ones.

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